develops [corresponding to the appearance of a spatially varying density (or charge) wave], and one gets a transition from a spatially homogeneous to a crystalline state.[6]

The above discussion serves to make explicit one of the central arguments given in BHP – that the ground state of H_{ext} would become degenerate only if the added field-particle interaction pushed down some excited states of H_{elec} to the ground state. They noted that "this would be highly implausible except possibly for pathological systems. There seems to be a general theorem in physics, so far unproved, which states that *when an external* (static) *field is applied to a physical system the reaction of the system will never completely cancel it.*" The electron liquid, at a density such that a charge wave develops spontaneously, represents just such a pathological system.

---

[1] D. Bohm and D. Pines, Phys. Rev. 92, 609 (1953).

[2] D. Bohm, K. Huang, and D. Pines, Phys. Rev. 107, 71 (1957).

[3] R. L. Coldwell, previous paper, Phys. Rev. B 2, 1423 (1970).

[4] See, for example, D. Pines and P. Nozières, *Theory of Quantum Liquids* (Benjamin, New York, 1966), pp. 95–99.

[5] See Ref. 4, pp. 206–209.

[6] See also P. C. Martin, Phys. Rev. 161, 143 (1967).

PHYSICAL REVIEW B VOLUME 2, NUMBER 5 1 SEPTEMBER 1970

Positronium Decay in Condensed Matter

Werner Brandt and James H. Fahs

Department of Physics, New York University, New York, New York 10003

(Received 11 March 1970)

In earlier papers [Phys. Rev. 120, 1289 (1960); 142, 231 (1966)] the pickoff annihilation rate of positronium was calculated in a cell approximation for various cell geometries. The rate can be expressed in terms of two functions $F(S, v^*)$ and $\xi(S, v^*)$ that depend on a cell potential parameter S and the reduced cell volume v^*. These functions are rederived and previous errors corrected. Extensive numerical tabulations are made available and analytical approximations presented.

The decay of positronium (Ps) in matter can be influenced strongly by electron pickoff. In this process the positron, bound to an electron as P_s annihilates with an electron bound in the medium. The pickoff annihilation rate is proportional to the overlap of the positron wave function with the electron wave function of the medium.

The pickoff annihilation rate Γ_p was derived for a simple cell model of condensed matter,[1,2] with the result

$$\Gamma_p(S,\ v^*,\ T) = \Gamma_{0p}\,\frac{1+G\{\exp[2\xi^2(S,\ v^*)\langle u^2(T)\rangle/r_0^2]-1\}}{1+F(S,\ v^*)}\ . \tag{1}$$

In Eq. (1), $\Gamma_{0p}=\pi r_e^2 c\rho_0$ is a constant proportional to the electron density ρ_0 in the domain of wave-function overlap; $S=(4m/\hbar^2)\,U_0 r_0^2$ is a dimensionless parameter proportional to the cell potential U_0 and the radius r_0 of the cell core; $v^*=v_1/v_0$ is the cell volume v_1 reduced by the volume v_0 of the cell core. The factor G varies between $\frac{1}{2}$ and 1, approximately as

$$G \simeq 1-\tfrac{1}{2}\tanh[(\mu-1)/\xi(S,\ v^*)]\ , \tag{2}$$

where $\mu=1,\ 2,\ 3$ for planar, cylindrical, and spherical cell symmetry, respectively. The dependence on temperature T of the reduced mean-square amplitude $\langle u^2\rangle/r_0^2$ of the core vibration can be expressed as

$$\langle u^2\rangle/r_0^2 = v^{*2/\mu}\gamma_M T/T_M\ , \quad T_D < T \leq T_M\ , \tag{3}$$

where γ_M is the critical value of $\langle u^2\rangle/r_0^2\ v^{*2/\mu}$ for which melting occurs; T_D is the Debye temperature, and T_M the melting temperature. In the liquid, we set

$$\langle u^2\rangle/r_0^2 \simeq v^{*2/\mu}\gamma_M, \quad T > T_M\ . \tag{4}$$

At temperatures $T < T_D$, $\langle u^2\rangle$ becomes a constant, and Γ_p depends only on S and v^*.

The formulas for the functions $F(S,\ v^*)$ and $\xi(S, v^*)$ are given in Table I for the three geometries, together with the auxiliary compatibility

TABLE I. Electron pickoff functions $F(S, v^*)$ and $\xi(S, v^*)$ and the compatibility relations for different cell geometries, with the abbreviations $\eta = (S-\xi^2)^{1/2}$ and $\eta_1 = \eta v^{*\,1/\mu}$, where $\mu = 1, 2, 3$ for planar, cylindrical, and spherical symmetries, respectively.

Geometry	Plane	Cylinder	Sphere
Compatibility	$\xi \tanh\xi + \eta \tan(\eta-\eta_1) = 0$	$\xi \frac{I_1(\xi)}{I_0(\xi)} + \eta \frac{G_1(\eta)}{G_0(\eta)} = 0,$ where $G_0(\eta) = Y_1(\eta_1)\,J_0(\eta) - J_1(\eta_1)\,Y_0(\eta),$ $G_1(\eta) = Y_1(\eta_1)\,J_1(\eta) - J_1(\eta_1)\,Y_1(\eta),$ $I_0,\ I_1$ = modified Bessel functions, $J_0,\ J_1$ = Bessel functions of first kind, $Y_0,\ Y_1$ = Bessel functions of second kind,	$\xi \coth\xi - 1 + \frac{(\eta_1\eta+1)\sin(\eta-\eta_1) - (\eta-\eta_1)\cos(\eta-\eta_1)}{\eta_1\cos(\eta-\eta_1) + \sin(\eta-\eta_1)} = 0$
$F(S, v^*)$	$\frac{\tan^2(\eta-\eta_1)}{\tanh^2\xi}\ \frac{1+2(\eta-\eta_1)/\sin 2(\eta-\eta_1)}{1+2\xi/\sinh 2\xi}$	$\frac{I_0^2(\xi)}{I_0^2(\xi) - I_1^2(\xi)}$ $\times\left[\frac{4/\pi^2\eta^2 - G_1^2(\eta)}{G_0^2(\eta)} - 1\right]$	$\frac{1}{2}\ \frac{\xi}{\eta}\ \frac{\cosh(2\xi)-1}{\sinh(2\xi)-2\xi}$ $\times \frac{(1-\eta_1^2)\sin 2(\eta-\eta_1) + 2\eta_1\cos 2(\eta-\eta_1) + (\eta_1-2\eta)(1+\eta_1^2) - \eta_1(1-\eta_1^2)}{\eta_1\sin 2(\eta-\eta_1) + 1 - (1-\eta_1^2)\cos^2(\eta-\eta_1)}$

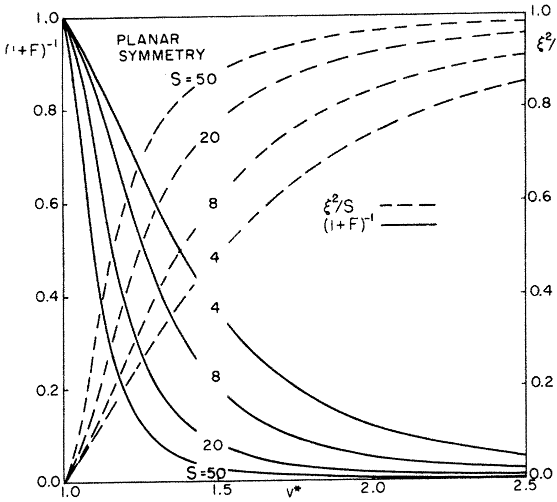

FIG. 1. The functions $F(S, v^*)$ and $\xi(S, v^*)$ for planar symmetry.

relations. Table I is an improved version of the table given in Ref. 1, in that the notation has been made more consistent and errors have been corrected.[3]

Numerical calculations have been made of F and ξ for the parameter ranges $1 \leq v^* \leq 2.5$ and $4 \leq S \leq 3.5\times10^3$ for $\mu = 1, 2,$ and $4 \leq S \leq 1.6\times10^3$ for $\mu = 3$. The tabulated results are available through the National Auxiliary Publication Service.[4] Sam-

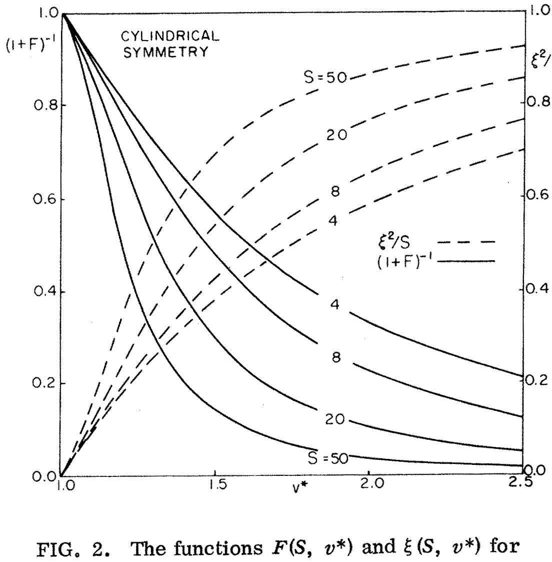

FIG. 2. The functions $F(S, v^*)$ and $\xi(S, v^*)$ for cylindrical symmetry.

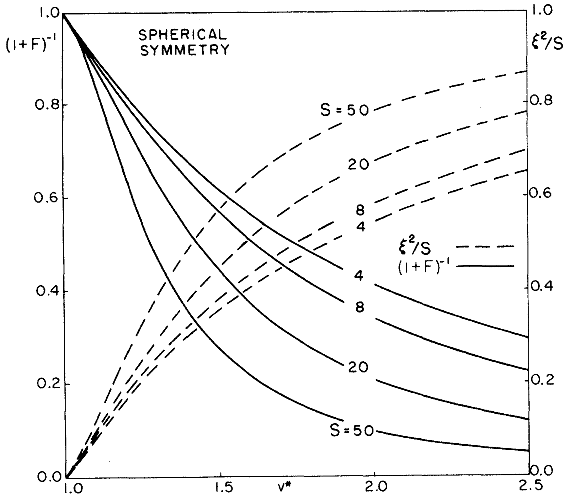

FIG. 3. The functions $F(S, v^*)$ and $\xi(S, v^*)$ for spherical symmetry.

ples for a range of S values are plotted in Figs. 1–3.

For many purposes, it is desirable and sufficient to use analytical approximations for these functions. The interpolation formulas

$$F(S,\ v^*) = (2/\pi)^2\{[1 + (v^{*1/\mu} - 1)\sqrt{S}]^3 - 1\} \tag{5}$$

and

$$\xi^2(S,\ v^*) = S[1 - (1 + F)^{-2/3}] \tag{6}$$

connect the asymptotic behavior at $v^{*1/\mu}\sqrt{S} \gg 1$ with the proper limit at $v^* = 1$.[5] When inserted in Eq. (1), they approximate the exact Γ_p to $\lesssim 20\%$ over parameter ranges of interest.

---

[1] W. Brandt, S. Berko, and W. W. Walker, Phys. Rev. 120, 1289 (1960).

[2] W. Brandt and I. Spirn, Phys. Rev. 142, 231 (1966).

[3] We are grateful to Dr. L. Zappa for correspondence on this point.

[4] W. Brandt and J. H. Fahs, *Positron Annihilation in Matter*: *Tables of Electron Pickoff Functions* (National Auxiliary Publication Service, 1970).

[5] Large values of the arguments v^* and S occur when these relations are applied to the electron pickoff from macroscopic particles [W. Brandt and R. Paulin, Phys. Rev. Letters 21, 193 (1968)].

PHYSICAL REVIEW B VOLUME 2, NUMBER 5 1 SEPTEMBER 1970

Knight Shift in Small Particles at Low Temperatures*

E. Philip Horvitz†

Rice University, Houston, Texas 77001

(Received 12 November 1969)

It is shown that the magnetic moments of the O_2 molecules in a surface layer of air adsorbed to small particles will create a static magnetic field at low temperatures that is large enough to affect the measured Knight shifts and linewidths.

It is the purpose of this note to point out that if a layer of air is adsorbed to the surfaces of small particles (dimensions on the order of hundreds of Angstroms or less) it will affect the measured Knight shifts and linewidths at low temperatures. This happens because the magnetic dipole moments of the O_2 molecules in the layer of air are sufficiently polarized at low temperatures that they create a magnetic field inside the particle that is in many cases on the order of the inherent Knight shift.

The static magnetic field $H(O_2)$ created inside a small particle by the O_2 molecules adsorbed on its surface will now be calculated. A typical particle is in the form of a cylindrical platelet. For the purpose of calculation the external field H_0 is in the x direction. The axis of the cylinder is along the x axis and the cylinder has a length l. The two circular disc ends of the cylinder are located at $x = 0$ and $x = l$ and have a diameter d. It is obvious that only the component of $H(O_2)$ in the x direction, $H_x(O_2)$, will contribute to an observed shift. It is further assumed that $H_x(O_2)$ does not vary in the plane perpendicular to the x axis. This is a reasonable assumption for the order-of-magnitude calculation that is being made. (For example, the field from the O_2 dipoles on the flat end